Theoretical Analysis of Mobile Operators' Spectrum Strategies

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Abstract: In this paper, I construct a mathematical model based on the Shannon-Hartley theorem and find profit-maximizing conditions for a mobile operator as for its channel bandwidth, the number of the channels, the S/N ratio, density of base stations in congested areas and the number of its subscribers. The following results and implications are obtained by the theoretical analysis. Firstly operators will fix their prices so that the price elasticity of demand can be one in the absence of congestion. However, once congestion arises, the optimum number of subscribers under the congested circumstances should be less than the number without congestion. Secondly, operators will choose their investment either in devices or in base stations to keep a throughput speed, so that the technical marginal rate of substitution can equal the ratio of the marginal costs. This result implies that operators may increase density of base stations in congested areas instead of ameliorating the network equipment. Thirdly, there is a difference between the marginal revenue and the marginal costs as for the bandwidth of the each channel, and this difference becomes larger as the bandwidth of each channel becomes narrower, and as the number of channels becomes more. Fourthly, the optimum channel bandwidth becomes narrower in general, if operators can choose both channel bandwidth and the number of channels. Finally, the spectrum cap per operator does not make sense in spectrum assignment. Either through spectrum auctions or through beauty contests, if the costs of acquisition of spectrum increase as the assigned bandwidth becomes larger, operators may use spectrum efficiently in the sense that they economize the bandwidth.

Key words: spectrum, Shannon-Hartley theorem, technical standards.

Spectrum Auctions take place in many countries for several telecommunications services. We have experienced some speculative auctions, while we feel that the successful bids were underestimated in others. My original research question concerns

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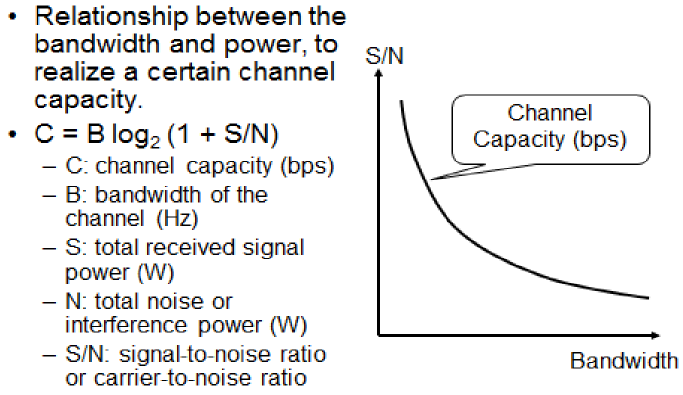
appropriate spectrum valuation of telecommunications operators and spectrum management authorities. Many economists have studied this problem from regression analysis and game theory. For example, P. Klemperer (KLEMPERER, 2002) has written one of the famous and classical papers in this subject. However, these analyses have implicitly regarded spectrum as the only input and neglected the operators' choices of the mobile technology.

The first economist who referred to substitution of the technology for spectrum resources was K. R. Carter (CARTER, 2009). He quoted the Shannon-Hartley theorem and served an idea to solve the valuation problem from two inputs model (figure 1). This theorem states the maximum channel capacity in bit per second (bps) that can be sent with a combination of a bandwidth of the channel in hertz (Hz) and a signal-noise (S/N) ratio over the bandwidth. The bandwidth and the S/N ratio correspond to the spectrum resources and the quality of equipment of the system respectively and there is a trade-off between these two inputs. Thus, the curve drawn by the theorem can correspond to an isoquant curve in the production theory in economics.

Figure 1 – Shannon-Hartley theorem



- Relationship between the bandwidth and power, to realize a certain channel capacity.
- $C = B \log_2 (1 + S/N)$
  - C: channel capacity (bps)
  - B: bandwidth of the channel (Hz)
  - S: total received signal power (W)
  - N: total noise or interference power (W)
  - S/N: signal-to-noise ratio or carrier-to-noise ratio

K.R. Carter ceased deepening the idea. However I developed and applied this idea to the spectrum valuation problem. Firstly I used a graphical presentation to show the value of spectrum drawn by the marginal rate of technical substitution (YUGUCHI, 2010). The value drawn by this method refers to only a value of a channel and does not concern value of a band of spectrum, so I constructed mathematical models to consider the whole band (YUGUCHI, 2011a, 2011b, 2012).

The application of the Shannon-Hartley theorem may be polemic both from the engineering viewpoints and from the economic viewpoints. The theorem was established more than sixty years ago and states the theoretical upper bound of capacity on error free transmission, so that it cannot fully reflect the actual technology and the environment in the mobile network construction such as an argument on the fifth-generation wireless systems (5G). The competition in the downstream markets (mobile communications market) is far more important than in upstream markets for the factors of production, because it is the downstream market that affects the consumer prices.

However, the trade-off between the bandwidth of the channel and the S/N ratio contains a serious choice problem for operators, spectrum authorities and regulators. To increase the S/N ratio, operators adopt technologies reducing the noise level, higher-order modulations, higher antenna power, etc. for their whole systems, i.e. networks and terminals, and/or dense networks of base stations in congested areas. These measures need generally high costs not only for the operators but also for the mobile users. Terminals adopting these technologies need batteries and/or filters of higher quality and then become expensive. In a country such as Japan where operators share largely the cost of terminals, the operators' costs become high if they tend to increase the S/N ratio. Higher antenna power must affect the guard bands and guard areas, and thus the spectrum authorities request the operators to improve their systems to keep smaller guard bands and guard areas. This regulation also invites higher costs for the operators. As a result, the operators should compare these costs with spectrum acquirement costs. While each operator demands a larger bandwidth, the total bandwidth available for the mobile communications is limited. Spectrum becomes more valuable, and then the operators face the trade-off not only in the engineering sense but also in the economic sense. The spectrum authorities end up deciding the competition in the downstream market of which the regulators are in charge.

It is important to consider both the spectrum problem and the technical choice problem at the same time. Investment in the network equipment is often larger than investment in spectrum. According to the securities report of NTT DoCoMo in the fiscal year 2011, which ended on 31st March 2012, NTT DoCoMo, the top mobile operator in Japan, has 7.3 trillion yen of property, plant and equipment, of which 6.5 trillion yen relates to the network infrastructure such as the base stations and the backbone. The annual depreciation amounts to 415 billion yen. On the other hand, the annual payment of taxes and public dues except the income tax reaches 40.6 billion yen. This payment contains not only the property tax but also the dues for spectrum. In Japan spectrum users should pay the annual "spectrum user fee," and the license application or reissuing fee in general every five years. NTT DoCoMo pays around 25.4 billion yen for the annual fee in the fiscal year 2010 . Thus, the spectrum cost (25.4 billion yen) is much smaller than the infrastructure cost (415 billion yen) for the mobile operators. We can easily guess that mobile operators prefer using more spectrum resources to upgrading the performance of their equipment for increasing the throughput speed.

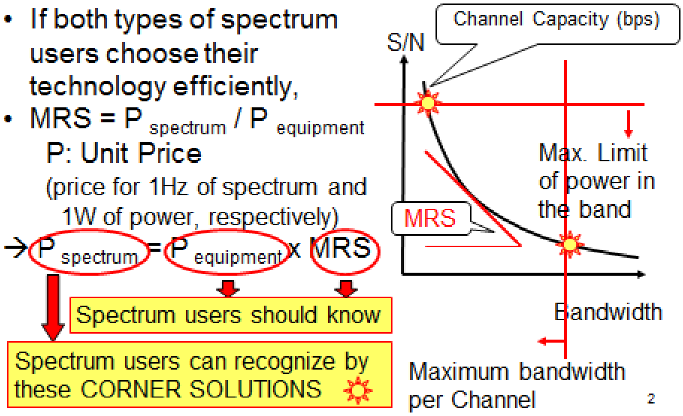
In this paper, I construct a mathematical model based on the Shannon-Hartley theorem and find profit-maximizing conditions for a mobile operator as for its channel bandwidth, the number of the channels, the S/N ratio, density of base stations in congested areas and the number of its subscribers. Here, apart from the spectrum valuation problem, we consider an operator's optimum investment problem and its spectrum strategy. This approach may be useful for examining the 5G, which has very recently become under discussion. In the 5G, an estimated real throughput speed will reach the theoretical maximum level defined by the Shannon-Hartley theorem, and to realize this speed, we must consider three dimensions, i.e. time, spectrum and geographical areas. The S/N ratio and the density correspond to the time and the area. I show briefly previous studies in the 2nd section, and construct a mathematical model in the 3rd section. After solving an operator's profit-maximizing conditions, I analyze the results in the 4th section and mention implications in the 5th section. Finally I end with concluding remarks in the last section.

■ Previous studies

CARTER (2009) was a pioneer, including in the Shannon-Hartley theorem in economic models.

YUGUCHI (2010) tried to find an appropriate value of spectrum by using corner solutions stemmed from regulatory constraints both on maximum power and a maximum bandwidth per channel (figure 2). Maximum power provides more or less the S/N ratio. In addition, mobile operators know the relationship between S/N ratios and bandwidths to realize a necessary channel capacity, and the network construction and maintenance costs. Thus, mobile operators should calculate an appropriate value of spectrum by the marginal rate of technical substitution (MRS) and the costs of network equipment. However, the Shannon-Hartley theorem refers only to the maximum speed of a channel. Normally, operators construct their network by bundling the channels. We should consider the whole bandwidth they acquired through either spectrum auctions or licensing processes, not only a single channel.

Figure 2 – Appropriate factor demand and the value of spectrum



- If both types of spectrum users choose their technology efficiently,
- MRS = $P_{\text{spectrum}} / P_{\text{equipment}}$

P: Unit Price

(price for 1Hz of spectrum and 1W of power, respectively)

→ $P_{\text{spectrum}} = P_{\text{equipment}} \times \text{MRS}$

Spectrum users should know

Spectrum users can recognize by these CORNER SOLUTIONS ☼

YUGUCHI (2011a) elaborated a mathematical model improving these weaknesses in my original model. YUGUCHI (2011b) developed my second model and included the channel capacity, the bandwidth assigned to an operator, the investment in base stations in congested areas in one mathematical model. The latter model was almost accomplished except for several elements on the assumptions of mathematical functions and its political implications. YUGUCHI (2012) revised these models and referred to some implications. However, I find that I did not make clear the results of the model and implications.

The models based on the Shannon-Hartley theorem have played an important role in the recent technological environment. In the theorem, a theoretical maximum speed i.e. channel capacity, depends on both the S/N ratio and the bandwidth of each channel. The whole bandwidth is equal to the channel bandwidth multiplied by the number of channels. Under the traditional technological standards, the channel bandwidth was defined by the technical parameters of each telecommunications system, and thus there was no room for choice of each operator. In other words, each operator could choose only the number of channels under its estimation of a future client size, once it decided its telecommunications system.

Under the new technical standards, such as the LTE (Long Term Evolution) and the WiMAX (Worldwide Interoperability for Microwave Access), operators can choose a channel bandwidth or variable channel bandwidths among several options. For example, in the LTE-Advanced, there are 6 options on channel bandwidths ("component carriers"): 1.4, 3, 5, 10, 15 or 20 MHz, and in addition, a maximum of five component carriers can be aggregated. Operators have great flexibility of combinations of channel bandwidths and S/N ratios, and can design appropriate parameters of their systems. It may result in change of spectrum assignment method both in spectrum auctions and in beauty contests. Spectrum assignment of a fixed and a large block will be replaced by a flexible amount of bandwidth.

On the other hand, technical restrictions such as a maximum power limit and a maximum acquisition bandwidth per each operator will still remain from spectrum managerial viewpoints. It is important for regulators and policy makers to know how seriously these restrictions may affect operators' flexible choices and the value of spectrum resources. At the same time, it is also important for them to know an operator's "natural" choices of technical parameters. In its profit-maximizing behavior, which parameters, i.e. bandwidth of each channel, the number of channels and density of base stations, will the operator choose? In the following section, I construct a mathematical model to answer this question.

■ Mathematical model

We suppose a monopolistic competitive market in telecommunications services. The number of licensed operators is limited by an assignable bandwidth. We cannot image a perfectly competitive market. Each operator can differentiate combinations of a lump sum or fixed monthly fee and a nominal maximum speed. As I define later, the nominal maximum speed is different from the real or effective speed by effects of congestion. Consumers choose an operator among competitive ones on the basis of their need for the quality of service, the subscription price, the brand image and so on. Here, to avoid complexity, we suppose that all consumers recognize the real or effective speed as the quality of service, and all operators set their price of data communication services at lump sum monthly fees. Consumers often tend to estimate each operator's real speed through the news by word of mouth. A consumer generally subscribes to one operator.

The Shannon-Hartley theorem determines a channel capacity as a result of an operator's choice of a combination of an S/N ratio and a bandwidth (MHz). A group of the combinations is interpreted as the isoquant curve between two inputs in the production theory in economics. The isoquant curve represented the channel capacity (bps). Two elements, i.e. the S/N ratio and the bandwidth (Hz), deciding the channel capacity, are replaced into the system (i.e. network equipment and terminals if necessary) and spectrum resources respectively. The slope of this isoquant curve is represented by the technical rate of substitution and should be tangent to iso-cost line (i.e. input price ratio). Here, the isoquant curve means the theoretical channel capacity, i.e. the nominal speed. The real speed depends not only on a provider's investment in the network equipment and the bandwidth for a channel but also on the number of channels, the density of base stations in congested areas, and the number of its subscribers. As is often the case with collective consumption goods, the real speed will not diverge from the nominal speed until the threshold defined with the number of subscribers.

Suppose that there are N telecommunications operators (i=1,…,N) in the monopolistic competitive market. An operator (i) provides the data communications service at an effective throughput speed of q_i M bits/s for a monthly lump sum fee (p_i). The number of subscribers of the operator i's service, d_i, can be defined by the demand function $d_i = D_i\ (q_i,\ p_i,\ q_{others},\ p_{others})$. q_{others} and p_{others} correspond to a set of the speeds and a set of the monthly lump sum fees provided by operators other than i, respectively. The operator i constructs a_i channels designed for a channel capacity of r_i Mbits/s. The channel capacity of r_i M bits/s can be realized by s_i in S/N ratio and b_i MHz in bandwidth, where a regular function $r_i = R_i\ (s_i,\ b_i)$ is defined by the Shannon-Hartley theorem. However, this channel capacity means simply a theoretical maximum speed. The effective speed of q_i depends on the number of subscribers (d_i) and the operator's investment in the base stations in congested areas (k_i). k_i can be defined as density of base stations.

We do not think about license-exempted network operators here. Licensed operators can choose the appropriate service levels by controlling their investment and the number of their subscribers. Thus, the effective speed of qi M bits/s is defined by

$$q_i = Q_i\ (R_i(s_i,\ b_i),\ a_i,\ k_i,\ d_i)$$

where the function Q_i is regular, and $\partial Q_i/\partial R_i > 0$, $\partial Q_i/\partial a_i > 0$, $\partial Q_i/\partial k_i > 0$,and $\partial Q_i/\partial d_i < 0$ if $d_i > d_i^*$, $\partial Q_i/\partial d_i = 0$ otherwise. d_i^* is a threshold decided by a_i and k_i, and means that congestion will occur beyond d_i^*. Here this threshold is externally given for simplicity.

We can also define the supply cost function as $c_i = C_i (R_i (s_i, b_i), k_i, (a_i \times b_i))$. This cost function is also assumed to meet regularity. The first and second terms concern the network construction costs and the third term concerns the spectrum acquisition costs. We assume that $\partial C_i/\partial R_i > 0$, $\partial^2 C_i/\partial R_i^2 > 0$, $\partial R_i/\partial s_i > 0$, $\partial^2 R_i/\partial s_i^2 < 0$, $\partial R_i/\partial b_i > 0$, $\partial^2 R_i/\partial b_i^2 < 0$, $\partial C_i/\partial k_i > 0$, $\partial^2 C_i/\partial k_i^2 > 0$, and $\partial C_i/\partial (a_i \times b_i) > 0$. The sign of $\partial^2 C_i/\partial (a_i \times b_i)^2$ is left undetermined, although $\partial C_i/\partial (a_i \times b_i)$ is the marginal cost of the band of spectrum, and is concerned with spectrum assignment methods. Note that the cost function does not contain the number of subscribers as a variant. In other words the marginal costs for subscribers are zero.

The operator i will then choose di so that it maximizes its profit $\pi_i = d_i \times p_i - c_i$. To solve the problem, we suppose the Nash Conjecture; q_{others} and p_{others} are externally given. The operator i will neither affect another operator's strategy nor be affected by others. Therefore, $p_i = P_i (q_i, d_i)$, where $\partial P_i/\partial q_i > 0$, $\partial^2 P_i/\partial q_i^2 < 0$, and $\partial P_i/\partial d_i < 0$.

$$\begin{aligned}\pi_i &= d_i \times p_i - c_i = d_i \times P_i (q_i, d_i) - C_i (R_i (s_i, b_i), k_i, (a_i \times b_i)) \\ &= d_i \times P_i (Q_i (R_i (s_i, b_i), a_i, k_i, d_i), d_i) - C_i (R_i (s_i, b_i), k_i, (a_i \times b_i))\end{aligned}$$

The operator i can choose appropriate levels as for the parameters d_i, s_i, b_i, k_i and a_i, subject to $a_i \times b_i \leqq W$, where W is the maximum bandwidth assigned to one operator and generally given by the authority. Thus, we have the following first order conditions:

$$\begin{aligned}\partial \pi_i / \partial d_i &= P_i (Q_i (R_i (s_i, b_i), a_i, k_i, d_i), d_i) \\ &+ d_i \times \{(\partial P_i / \partial Q_i) \times (\partial Q_i / \partial d_i) + (\partial P_i / \partial d_i)\} = 0 \qquad [1]\end{aligned}$$

$$\partial \pi_i / \partial s_i = d_i \times \{(\partial P_i / \partial Q_i) \times (\partial Q_i / \partial R_i) \times (\partial R_i / \partial s_i)\} - \{(\partial C_i / \partial R_i) \times (\partial R_i / \partial s_i)\} = 0 \qquad [2]$$

$$\partial \pi_i / \partial k_i = d_i \times \{(\partial P_i / \partial Q_i) \times (\partial Q_i / \partial k_i)\} - (\partial C_i / \partial k_i) = 0 \qquad [3]$$

$$\begin{aligned}\partial \pi_i / \partial b_i &= d_i \times \{(\partial P_i / \partial Q_i) \times (\partial Q_i / \partial R_i) \times (\partial R_i / \partial b_i)\} \\ &- \{(\partial C_i / \partial R_i) \times (\partial R_i / \partial b_i)\} - a_i \times \{\partial C_i / \partial (a_i \times b_i)\} - \lambda a_i = 0 \qquad [4]\end{aligned}$$

$$\partial \pi_i / \partial a_i = d_i \times \{(\partial P_i / \partial Q_i) \times (\partial Q_i / \partial a_i)\} - b_i \times \{\partial C_i / \partial (a_i \times b_i)\} - \lambda b_i = 0 \qquad [5]$$

λ is a Lagrangian coefficient.

■ Results

The profit maximum conditions stemmed from these five equations above may imply the following facts.

At first, the equation [1] refers to the optimal number of subscribers. We can rewrite this formula as

$$\{P_i (Q_i (R_i(s_i, b_i), a_i, k_i, d_i), d_i) + d_i \times (\partial P_i / \partial d_i)\} + \{d_i \times (\partial P_i / \partial Q_i) \times (\partial Q_i / \partial d_i)\} = 0 \qquad [1']$$

The first part is the marginal revenue for the subscribers, and the second part concerns the congestion effects which is $\partial Q_i/\partial d_i < 0$ if $d_i > d_i^*$ or 0 otherwise. In the absence of congestion, $p_i + d_i \times (\partial P_i / \partial d_i) = 0$. It means that the marginal revenue equals 0, and that the optimum price can be fixed so that the price elasticity of demand becomes 1. However, once congestion arises, the second term of the equation [1'] becomes negative. This implies that the optimum number of subscribers in the congested circumstances should be less than the optimum number without congestion.

Secondly, the equations [2] and [3] indicate that additional revenue from the improvement of the equipment (S/N ratio) and the network (density of base stations) should be equal to the additional costs of this improvement. These two equations are banal in the production theory. However, after transformation of these two equations, we have the following results;

$$\{d_i \times (\partial P_i / \partial Q_i) \times (\partial Q_i / \partial R_i)\} = (\partial C_i / \partial R_i) \qquad [2']$$

$$\{d_i \times (\partial P_i / \partial Q_i) \times (\partial Q_i / \partial k_i)\} = (\partial C_i / \partial k_i) \qquad [3']$$

Thus, we have $(\partial Q_i/\partial R_i) / (\partial Q_i / \partial k_i) = (\partial C_i / \partial R_i) / (\partial C_i / \partial k_i)$. It is clear that the operator will choose its investment either in equipment or in base stations along the condition that the technical marginal rate of substitution equals the ratio of the marginal costs.

Thirdly, the equations [4] and [5] relate to the operator's strategy for spectrum. Effacing the Lagrangian coefficient λ and arranging these two equations, we derive the following equation.

$$d_i \text{ x } \{(\partial P_i / \partial Q_i) \text{ x } (\partial Q_i / \partial R_i) \text{ x } (\partial R_i / \partial b_i)\} - (a_i / b_i) \text{ x } \{d_i \text{ x } (\partial P_i / \partial Q_i) \text{ x } (\partial Q_i / \partial a_i)\} = (\partial Ci / \partial Ri) \text{ x } (\partial Ri / \partial bi) \qquad [6]$$

The first term of the left side means marginal revenue from amelioration of the quality through enlargement of the bandwidth in each channel. The second term is positive and concerns amelioration of the quality through increase in the number of channels. The right side of the equation means marginal costs of enlargement of the bandwidth in each channel. The equation [6] can be rewritten by

$$d_i \text{ x } \{(\partial P_i / \partial Q_i) \text{ x } (\partial Q_i / \partial R_i) \text{ x } (\partial R_i / \partial b_i)\} - (\partial C_i / \partial R_i) \text{ x } (\partial R_i / \partial b_i)$$
$$= (a_i / b_i) \text{ x } \{d_i \text{ x } (\partial P_i / \partial Q_i) \text{ x } (\partial Q_i / \partial a_i)\} \qquad [6']$$

Concerning the bandwidth of each channel, the difference between marginal revenue and marginal costs can be affected by the number of channels and the bandwidth of each channel. The difference becomes larger as the bandwidth of each channel is narrower, and as the number of channels becomes more. Such a tendency reflects high demand for the operator's service. If the operator cannot choose the number of channels ($(\partial Q_i / \partial a_i) = 0$) against the initial assumptions, the right side of the equation [6'] is 0, but, in contrast, if it can control both the bandwidth of each channel and the number of channels at the same time, the right side is positive. Thus, the difference between marginal revenue and marginal costs is positive. It means that the operator can earn more revenue than the marginal costs by simultaneously controlling two parameters. By the initial presumption above ($\partial^2 P_i/\partial q_i^2 < 0$ and $\partial^2 C_i/\partial R_i^2 > 0$), we can conclude that the operator will prefer narrower bandwidth for each channel to increase the number of channels, when it can control both parameters at the same time. Note that the equation (6') is the same, even if we do not consider the constraint on the maximum bandwidth assigned by the authority to each operator.

Finally we should consider the sign of $\partial^2 C_i / \partial(a_i \text{ x } b_i)^2$. If $\partial^2 C_i / \partial(a_i \text{ x } b_i)^2 > 0$, the marginal value of spectrum becomes higher as the bandwidth assigned to one operator becomes larger. Otherwise, the marginal price or value of spectrum becomes lower as the bandwidth becomes larger. The sign of this term has not taken an important role in this present model, although it is a very interesting issue both for our research and the spectrum economics.

■ Implications

The mathematical model presented above may imply several implications in spectrum management.

Firstly operators will fix their prices so that the price elasticity of demand can be one in the absence of congestion. However, once congestion arises, the optimum number of subscribers under the congested circumstances should be less than the number without congestion. It is true that this result does not attract our interests. In this model, the congestion effect is embodied. Thus, operators tend to behave as if they prevent their service from crowding in order to keep their lump sum fees high. Without congestion effect, however, they will allow some congestion, because they need a huge investment in the network infrastructure to respond to the congestion problem. In this sense, this model states explicitly a relationship between restraints of the subscribers and a need of the additional investment. Probably the operators will realize this solution through increase in their prices or deterioration in relative service levels among competitive operators. By the way, what is the meaning of the price elasticity of demand equal to one? Price strategy is null as for the revenues of the operators. This result stems from cost neutrality of the number of subscribers in the assumption. In this model, the congestion is the only key factor.

Secondly, operators will choose its investment either in equipment or in base stations to keep a throughput speed, so that the technical marginal rate of substitution can equal the ratio of the marginal costs. This result implies that operators may increase density of base stations in congested areas instead of improving the network devices through enlargement of bandwidth of each channel or improvement of the S/N ratio.

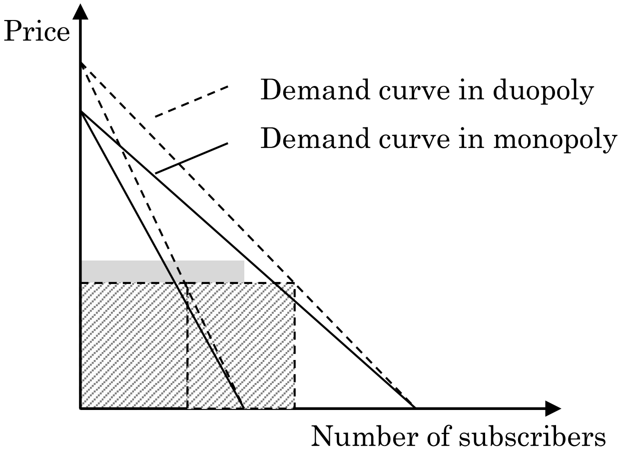
Thirdly, there is a difference between the marginal revenue and the marginal costs as for the bandwidth of each channel, and this difference becomes larger as the bandwidth of each channel is narrower, and as the number of channels becomes more.

Fourthly, the optimum channel bandwidth becomes narrower in general, if operators can choose both channel bandwidth and the number of channels.

Finally, the spectrum cap per operator does not make sense in spectrum assignment. Either through spectrum auctions or through beauty contests, if the costs of acquisition of spectrum increase as the assigned bandwidth becomes larger, operators may use spectrum efficiently in the sense that they economize the bandwidth.

The result concerning the spectrum cup is somewhat surprising and controversial. It is sure that a lack of the spectrum cup may result in a monopolistic market in the downstream mobile market. However, such a situation must invite a speculative price for spectrum and potential monopolies must refrain from obtaining a huge bandwidth. As a result, the potential monopolies are interested in improvement of the S/N ratio. As the number of operator increases, the spectrum price becomes dramatically less and the operators tend to acquire a wide bandwidth. Without spectrum cap, the factor market works well in restraining the birth of the monopoly in the downstream market.

Figure 3 – Downstream market



A simple figure (figure 3) may explain this situation. In this model, the marginal cost for subscribers is zero. Under the linear demand schedule for the mobile communications, potential monopolies will fix a lump sum price in order to supply for a half of the potential subscribers or differentiated prices in order to supply for all the potential subscribers, according to a regulation in the downstream market. Thus, the spectrum price may be fixed at most at the monopolistic profit equal to the revenue minus the fixed costs. If we assume a duopolistic market composed of two companies with the same cost structure and the same strategy, potential duopolies will decrease a lump-sum price in order to supply for a third of the potential subscribers. Thus, the spectrum price may be fixed at most at the duopolistic profit equal to the revenue minus the fixed costs. This spectrum price is dramatically less than the price in a monopolistic market, due to the dual investment in the network infrastructure, even if the competition in the downstream market leads amelioration of mobile services and then increases the potential subscribers' willingness-to-pay.

■ Concluding remarks

In this paper, I elaborated a new mathematical model to analyze mobile operators' "natural" choices for both their technical parameters such as the bandwidth of the channel, the S/N ratio, the density of base stations in congested areas, and their managerial parameters such as the number of channels and subscribers. This mathematical model can show us the "natural" technical choices and spectrum strategy of mobile operators at the same time, and should be very useful for further analysis in regulators' spectrum management policy. Here, we should pay attention to definition of the cost function; the cost function remained neutral regardless of the number of subscribers. This assumption stemmed from an industry with a heavy infrastructure.

For further studies, I should firstly revise the cost function so that the function can reflect the number of subscribers, and compare the results between two cases. Secondly I should compare theoretical results with the actual situation, because my analysis stopped at the theoretical level. For the present, the technology with a great flexibility of parameters is limited to the fourth (including the 3.9) generation of wireless systems. Thus, I have to wait for the results of the licensing process of these standards in many countries. As I suggested in the introduction, all of the mobile operators in Japan request the regulatory body (Ministry of Internal Affairs and Communications) to allow constructing their networks with a larger channel bandwidth. International comparison of operators' choices and strategies may be very interesting from the regulatory point of view i.e. with and without spectrum auction.

With technological advances in wireless communications, network operators' technical flexibility may be enlarged. Thus, we need more sophisticated analytical models to evaluate operators' strategies and political tools from technological viewpoints.

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