An Investment Criterion Incorporating Real Options

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Abstract: Investment in infrastructure such as the information and communication technology sector requires large, substantial amounts, most of which are sunk or irreversible. Uncertainty of market demand, competition, costs and public policy complicates the investment decision process. This paper provides an investment decision-making criterion under uncertainty using (deferred) real options methodology to evaluate if an investment should be made immediately, cautiously, deferred (wait-and-watch), or foregone. A decision-making index d is developed, which is equal to the expectation of net present value (NPV) normalized by its standard deviation. Under a lognormal assumption of the distribution of NPV discounted by risk-free rate, we find the "break-even point" at which the NPV equals the real option value (ROV): d = D* = 0.276. Using the absolute value of D*, one can make sophisticated decisions considering opportunity losses. This new decision index, d, provides a criterion to make investment decisions to capture underlying uncertainty. When making a decision, a manager only has to observe three parameters: expectation of future cash flow, its uncertainty as measured by its standard deviation, and the magnitude of investment. We discuss examples using this criterion and show its value. The criterion is particularly useful when NPV lies near zero or uncertainty is large.

Key words: Real Options, Decision, Investment, Economic Methodology; Statistical Decision Theory, Criteria for Decision-Making under Risk and Uncertainty.

Investment in the information and communication technology (ICT), as with other infrastructure investments, requires large, substantial amounts, many of which are sunk or irreversible (ALLEMAN & RAPPOPORT, 2006; PINDYCK, 2004, 2005a). Thus, the decision to invest is critical, since it cannot be reversed without a significant loss. The uncertainty of future environment, including market demand, competition, costs and public policy further complicates the decision. In this paper we develop a decision-making criterion using real options methodology to capture the underlying uncertainty to evaluate a potential investment. This criterion provides guidance as to whether an investment should be made immediately, cautiously, deferred (wait-and-watch), or foregone. An index d is developed, which is equal to the expectation of net present value (NPV) normalized by its standard deviation.

By determining where the distribution of NPV discounted by risk-free rate is equal to the value of real option (ROV) to defer, we find the critical value of the index d. Under the lognormal distribution the critical value of d is equal to 0.276. Comparing this value to the calculated value of the "normalized" NPV, one can make decisions whether to invest considering opportunity losses. When making a decision, a manager only has to observe three parameters: expectation of future cash flow, its uncertainty as measured by its standard deviation, and the magnitude of investment. We discuss examples using this criterion. The criterion is particularly useful when NPV lies near zero or uncertainty is large.

This paper is divided into a discussion of financial options, which is the foundation of the real options methodology. The next section briefly discusses real options. The first two sections lay the foundation for the development of the decision-making index and develop the rules to apply this index to investment decisions. The last section, prior to the Conclusion, provides examples to the application of the techniques.

■ Financial options [1]

A financial option is the right to buy (a call) or sell (a put) a stock, but not the obligation, at a given price within a specific period of time. Several approaches are possible to determine the theoretical value of an option, based on different assumptions concerning the market, the dynamics of stock price behavior, and individual preferences. Option pricing theory is based on the no-arbitrage principle, which is applied to the underlying stock's distributional dynamics. The simplest of these theories is based on the multiplicative, binomial model of stock price fluctuations, which is often used for modelling stock behavior.

1 This section can be skipped by those who are familiar with option pricing theory.

The binomial model [2]

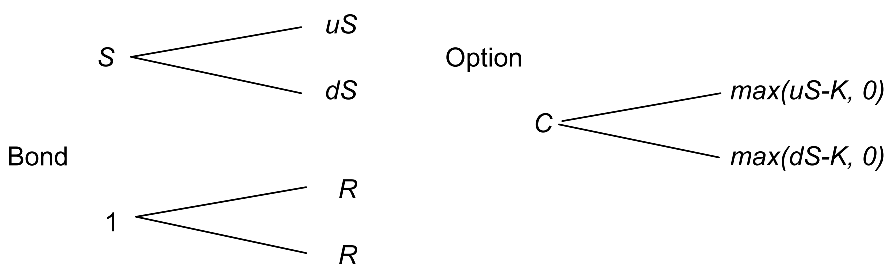
Assume a stock trades at a price S. Within one period, the price will be either uS or *dS*. Further assume we have a risk-free bond with return $R = 1+r_f$ (where r_f is the risk-free rate of interest) per period. To avoid arbitrage opportunities, we must have:

$$u > R > d \qquad [1]$$

If a stock option confers the right to buy the stock at the price K, called the exercise price or strike price one period later, the payoffs of the option are shown in equation [1.2] and Figure 1.1.

$$\begin{aligned} C_u &= \max(uS - K, 0) \\ C_d &= \max(dS - K, 0) \end{aligned} \qquad [1.2]$$

Figure 1.1



One can purchase *x* dollars worth of stock and *b* dollars worth of the bond. One period later, this portfolio will be worth either *ux* + *Rb* or *dx* + *Rb*, depending on the whether the outcome was the upper or lower path. To match the option outcomes, therefore, requires:

$$\begin{aligned} ux + Rb &= C_u \\ dx + Rb &= C_d \end{aligned} \qquad [1.3]$$

Solving this equation:

$$x = \frac{C_u - C_d}{u - d}$$

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[2] This interpretation is from LUENBERGER (1998). For an in-depth understanding of options see HULL (2009).

$$b = \frac{C_u - ux}{R} = \frac{uC_d - dC_u}{R(u-d)}$$

Combining these, the value of the portfolio is:

$$x + b = \frac{C_u - C_d}{u-d} + \frac{uC_d - dC_u}{R(u-d)}$$

$$= \frac{1}{R}\left(\frac{R-d}{u-d}C_u + \frac{u-R}{u-d}C_d\right)$$

The value $x + b$ must be the value of the call option C because the payoffs of this portfolio are exactly the same as that of the stock option (The no-arbitrage principle or one-price principle). The portfolio made up of the stock and bond that duplicates the payoff of the option is referred to as a replicating portfolio.

$$C = \frac{1}{R}\left(\frac{R-d}{u-d}C_u + \frac{u-R}{u-d}C_d\right) \quad [1.4]$$

A simplified way to view this equation [1.4] is to define the quantity:

$$q = \frac{R-d}{u-d} \quad [1.5]$$

and from the relation $u > R > d$ assumed earlier, it follows that $0 < q < 1$. Hence q can be viewed as a probability and is referred to as the risk-neutral probability. Rewriting [1.4] yields [1.6]:

Option pricing formula 1

The value of a one-period call option on a stock governed by a binomial lattice process is:

$$C = \frac{1}{R}\left(qC_u + (1-q)C_d\right) \quad [1.6]$$

Another method to obtain this risk-neutral probability is found by solving the equation,

$$S = \frac{1}{R}\left(quS + (1-q)dS\right)$$

[1.6] can be written as:

$$C(T-1) = \frac{1}{R}\hat{E}[C(T)] \quad [1.7]$$

Here *C(T)* and *C(T-1)* are the option values at *T* and *T-1*, respectively, and $\hat{E}$ denotes expectation with respect to the risk-neutral probabilities.

This solution method can be extended to multi-period (*T*) options using the formula:

$$C = \frac{1}{R_T}\hat{E}[C(T)] \quad [1.8]$$

where R_T is the risk-free return to the time to expiration.

The option price is calculated using payoffs for all cases, using the risk-neutral probability in the expectation function and discounting with the risk-free rate.

The continuous additive model

Next, the one period continuous additive model is developed.

$$S_1 = RS_0 + Ru \quad [1.9]$$

where

S_t : stock price at time *t*

u : normal distribution with mean 0, variance σ^2

R : Return of risk-free asset

This model satisfies risk-neutral condition because

$$E[S_1] = E[RS_0 + Ru]$$
$$= E[RS_0] + E[Ru]$$
$$= RS_0$$

$$\text{i.e. } S_0 = \frac{1}{R}E[S_1] \quad [1.10]$$

A call option on this stock with exercise price = *RK* at time 1, payoff of option C(1) is:

$$C(1) = [\max(S_1 - RK, 0)]$$

i.e. $C(1) = 0$, if $S_1 < RK$

$$C(1) = S_1 - RK \text{, if } S_1 > RK \qquad [1.11]$$

This option price can be calculated using the general option pricing formula [1.8].

$$C = \frac{1}{R_T}\hat{E}[C(T)]$$

$$= \frac{1}{R}\hat{E}[C(1)]$$

$$= \frac{1}{R}\int_{-\infty}^{+\infty} C(1) f(S_1) dS_1$$

where $f(S_1)$ is probability density function of S_1, normally distributed.

$$= \frac{1}{R}\int_{RK}^{+\infty} (S_1 - RK) f(S_1) dS_1$$

substituting $S_1 = RS_0 + Ru$, $f(S_1) = (1/R) f(u)$, $dS_1 = Rdu$

$$= \frac{1}{R}\int_{K-S_0}^{+\infty} \left[(RS_0 + Ru - RK)\frac{1}{R} f(u) \right] Rdu$$

$$= \int_{K-S_0}^{+\infty} [(S_0 + u - K) f(u)] du \qquad [1.12]$$

Assume $V = S_0 + u$ is normally distributed with mean S_0, variance σ^2, then [1.12] can be rewritten as a general option pricing formula for continuous outcome model.

Option pricing formula 2

The value of a one-period call option on a stock governed by a continuous additive model is:

$$C = \int_{K}^{+\infty} (V - K) f(V) dV \qquad [1.13]$$

where *V*: the present value of the future random value discounted by risk-free rate,

K is the present value of exercise price, discounted by the risk-free rate.

This option formula will be used in the 3rd section.

Note that when *V* takes a lognormal distribution, this formula is equivalent to Black-Scholes call option formula (HERATH & PARK, 2001).

Following the discussion of the previous section, this model can be extended to multi-period (*T*) options using the model:

$$S_t = RS_{t-1} + R^t u_t \qquad [1.14]$$

where u_i is the random variable normally distributed with mean 0, variance σ^2

$$\text{i.e. } S_T = R^T S_0 + R^T \sum_{i=1}^{T} u_i \qquad [1.15]$$

This can be confirmed to be risk-neutral because:

$$E[S_T] = E[R^T S_0] + E\left[R^T \sum_{i=1}^{T} u_i\right] = R^T S_0$$

since $E[u_i] = 0$ for all *I*

$$\text{i.e. } S_0 = \frac{1}{R^T} E[S_T] = PV[E[S_T]]$$

And assuming u_i is not correlated with any other $u_{j \neq i}$, the variance of *PV*[*E*[S_T]] is:

$$\sigma_T^2 \equiv Var[PV[S_T]] = Var\left[\sum_{i=1}^{T} u_i\right] = T\sigma^2 \qquad [1.16]$$

or

$$\sigma = \frac{\sigma_T}{\sqrt{T}} \qquad [1.17]$$

The one-period standard deviation can be calculated from multi-period one. The standard deviation of the yearly return is called volatility [3]. When the return is defined as $PV[S_T / S_0]$, its volatility is given by (σ / S_0).

Although the option pricing theory has been developed in order to value financial options, it can be applied to the real asset or firm's project. The methodology is called "real options". The next section shows how it is applied.

■ Real options

Real Options methodology is an approach used to evaluate alternative management strategies using traditional option-pricing theory applied to the real assets or projects. For example, when managers decide to assess a new project, they face several choices beyond simply accepting or rejecting the investment. Other choices include delaying the decision until the market is favorable, or deciding to start small and expanding later if the result seems to be superior. The traditional valuation method, DCF analysis, fails to account for these other choices. The list of these real options is shown in Table 2.1 (ALLEMAN & RAPPOPORT, 2002) [4]

Table 2.1 – Description of options

Option	*Description*
Defer	To wait to determine if a "good" sate-of-nature obtains
Abandon	To obtain salvage value or opportunity cost of the asset
Shutdown and restart	To wait for a "good" state-of-nature and re-enter
Time e-to-build	To delay or default on project – a compound option
Contract	To reduce operations if state-of-nature is worse than expected
Switch	To use alternative technologies depending on input prices
Expand	To expand if srtae-of-nature is better than expected
Growth	To take advantage of future, interrelated opportunities

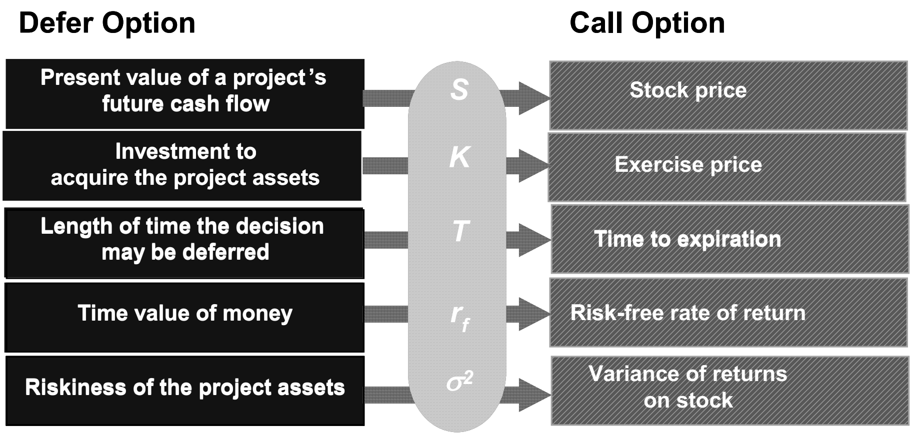
3 A precise definition of volatility is "the standard deviation of the return provided by the asset in one year when the return is expressed using continuous compounding" (HULL, 2009).

4 Methods to valuate each kind of option are described in several books TRIGEORGIS (1996), AMRAM & KULATILAKA (1999), COPELAND & ANTIKAROV (2001), and COPELAND, KOLLER & MURRIN (2000). Also see ALLEMAN, MADDEN & KIM (2008) in this issue for additional references.

Example: value of the option to defer

One real option alternative is the deferral option which is based on the concept of the call option, as shown in Figure 2.1 (LUEHRMAN, 1998a).

Figure 2.1



Consider a project, which is not currently profitable. A deferral option gives one the alternatives to defer starting this project for one year to determine if the price increases enough to make the investment worthwhile. This right can be interpreted as being similar to a call option. The numerical example illustrates its value.

Table 2.2 displays the project's present value of future cash flow. *V* is assumed to be normally distributed with mean \$100 million (= *S*) and standard deviation \$30 million (= σ). The risk-free rate is 6% (R=1.06); the exercise price one year later is \$110 million. With these assumptions, the project's present value is \$103.8 or \$110/1.06.

Table 2.2 – A project

Defer option	*Variable*	
Present value of operating future cash flow	S	\$100 milion
Investment in equipment	K	\$103.8 million
Length of time the decision may be deferred	T	1 year
Risk-free rate	r_f	1.06
Riskiness	σ	\$30 million

Conventional NPV is given by *S-K* = 100 – 103.8 = - 3.8 million. This project would have been rejected under NPV criterion. However, applying call option formula in the pricing equation [1.13], the value of deferring the project one year is calculated as the defer *ROV* (Real Option Value):

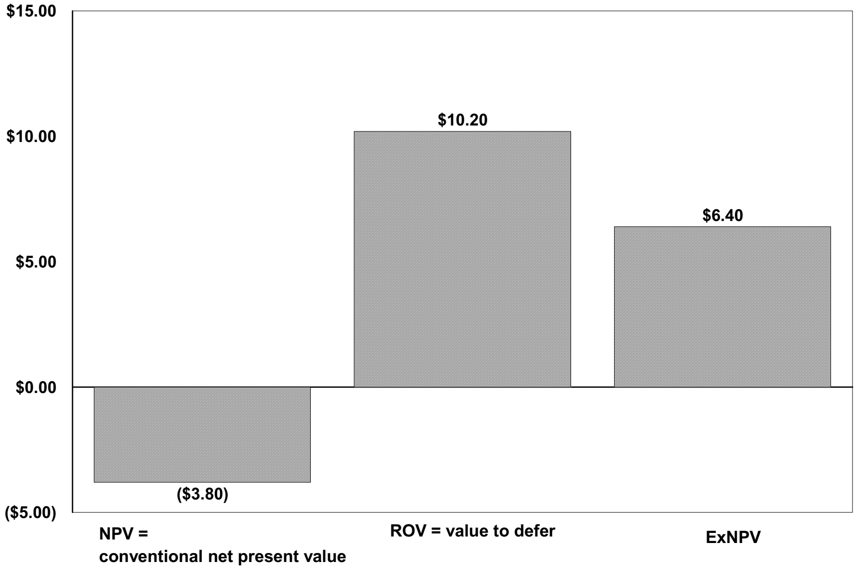
$$ROV = C = \int_{K}^{+\infty} (V - K) f(V) dV$$

$$= \int_{K}^{+\infty} (V - K) \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} \frac{(V - S)^2}{\sigma^2} \right) dV$$

$$= \int_{103.8}^{+\infty} (V - 103.8) \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} \frac{(V - 100)^2}{30^2} \right) dV = 10.2 \qquad [2.1]$$

The flexibility to defer this project is valued at 10.2 million.

Figure 2.2 – Expanded Net Present Value (ExNPV)



Adding NPV and ROV gives a positive value of 6.4 (= –3.8 + 10.2), see Figure 2.2. This is called Expanded NPV or *ExNPV*. *ExNPV* represents the value of this project including future flexibility (TRIGEORGIS, 1996). Consequently, the optimum decision is to "defer", i.e. "wait and watch the market!"

■ The new criterion

Decision under conditions of uncertainty should be made on the basis of the current state of information available to decision makers. If the expectation of the *NPV* were negative for the investment, the conventional approach would be to reject the investment. However, if one has the ability to delay this investment decision and wait for additional information, the option to invest later has value. This implies that the investment should not be undertaken at the present time. It leaves open the possibility of investing in future periods.

For the purpose of analyzing the relationship between NPV and the option value associated with the single investment, we assume that the random variable of interest is the present value of future cash flow *V*, which is assumed to be normally distributed $V \sim N(m', \sigma')$, where m' is the expected value of the discounted present value, E[V]. The investment cost *I* is assumed to be a constant.

In the conventional method, *NPV* is expressed as:

$$\begin{aligned} NPV &= E\,[V - I] \\ &= E[V] - I \\ &= m' - I \end{aligned} \qquad [3.1]$$

Two cases are examine: $(a_1, a_2) \in A$ are defined as Action 1: (a_1) do not invest when $NPV < 0$, and Action 2: (a_2) invest when $NPV > 0$.

Case 1: Action 1. Do not invest as *NPV* < 0

Here, following HERATH & PARK (2001), a loss function is introduced. When no investment takes place, obviously, the cash flow is equal to 0. But imagine the situation that $V > I$, where the opportunity loss is recognized as $V - I$. Therefore the loss function of Action 1 is:

$$\begin{aligned} L(a_1, V) &= 0 \quad \text{if } V < I \\ &= V - I \quad \text{if } V > I \end{aligned} \qquad [3.2]$$

The expected opportunity loss can be calculated as:

$$E[L(a_1,V)] = \int_{-\infty}^{+\infty} L(a_1,V) f(V) dV$$

$$= \int_{I}^{+\infty} (V-I) f(V) dV \qquad [3.3]$$

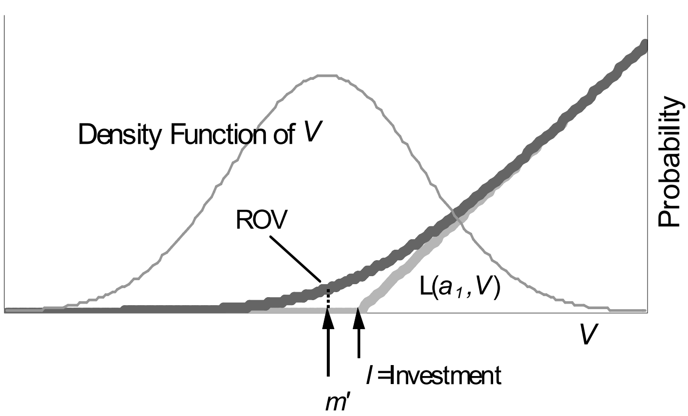
This function is the payoff of a call option, [1.11] using the pricing formula of a call option discussed in the previous section, [1.13].

Assuming an investment can be deferred until new information is obtained, this value is the same as the defer option for the investment. Moreover, the value is also equal to the expected value of perfect information (EVPI) for this investment opportunity (HERATH & PARK, 2001).

$$\text{ROV} = \int_{I}^{+\infty} (V-I) f(V) dV \qquad (3.4)$$

A similar relationship exists in Figure 3.1 as in Figure 1.2.

Figure 3.1 – Opportunity Loss function and ROV (NPV<0)



When the terminal distribution of V is normal, the real option value can be calculated using the unit normal linear loss integral:

$$L_N(D) = \int_{D}^{+\infty} (V-D) f_N(V) dV \qquad [3.5]$$

where $f_N(V)$ is the standard normal density function [5]

$$\text{ROV} = \sigma' L_N(D) \qquad [3.6]$$

$$\text{where } D = \frac{|m' - I|}{\sigma'}$$

When a manager makes an investment decision, her optimal decision is not to invest if $NPV = m'\text{-}I < 0$. Then she may compare the *NPV* and the defer option value. If she finds that the option value is larger than the absolute value of *NPV* (= | $m'\text{-}I$ |), she has the option to defer and watch for positive changes in the investment opportunity. If the option value is too small to compensate the *NPV* (< 0), she will abandon this investment proposal.

Decision Criterion 1: (Case of NPV < 0)

ROV > |*NPV*| → Wait and watch the opportunity carefully

ROV < |*NPV*| → Do not invest

We can solve the equation,

$$ROV = |NPV| \qquad [3.7]$$

$$\text{for } D = \frac{|m' - I|}{\sigma'}.$$

From [3.1] and [3.6], [3.7] is expressed as

$$\sigma' L_N(D) = |m' - I| \qquad [3.8]$$

Divided byσ', we have: (because $\sigma' > 0$)

$$L_N(D) = \frac{|m' - I|}{\sigma'}$$

$$\text{i.e. } L_N(D) - D = 0 \qquad [3.9]$$

Solving this equation, *D* is:

$$L_N(D) - D = 0$$

5 This expression is only for the case NPV < 0 though general expression is possible.

$$\int_{D}^{+\infty}(V-D)f_N(V)dV-D=0$$

$$\int_{D}^{+\infty}Vf_N(V)dV-D\int_{D}^{+\infty}f_N(V)dV-D=0$$

$$\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}D^2\right)-D\Phi(-D)-D=0$$

where $\Phi(a)=\int_{-\infty}^{a}f_N(x)dx$

$\therefore$ D = 0.276 [3.10]

And also the left hand side of equation [3.9] is decreasing as *D* increases because:

$\frac{d}{dD}(L_N(D)-D)<0$ for all $D > 0$ [3.11]

Decision Criterion 1': (Case of NPV < 0)

From [3.10] and [3.11], we can confirm that former Decision Criterion 1 can be written as:

$D < D^* \rightarrow$ Wait and watch the opportunity carefully

$D > D^* \rightarrow$ Do not invest

Case 2: Action 2. Invest as *NPV* > 0

Similarly, in this opposite case the loss function following HERATH & PARK (2001) is:

$L(a_2,V)=I-V$ if $V < I$ [3.12]

$= 0$ if $V > I$

The expectation of loss is given by:

$$E[L(a_2,V)] = \int_{-\infty}^{+\infty}L(a_2,V)f(V)dV$$

$$= \int_{-\infty}^{I} (I - V) f(V) dV \qquad [3.13]$$

These equations are similar to the payoff and price of the put option (Figure 3.2). Assuming we can defer this investment, the option value is the same as the expectation of the loss.

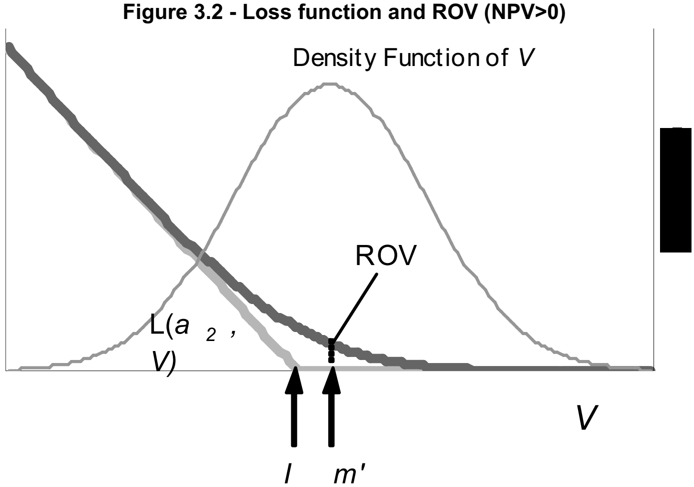
$$ROV = \int_{-\infty}^{I} (I - V) f(V) dV \qquad [3.14]$$

By symmetry, [3.14] can be written as:

$$ROV = \sigma' L_N(D) \qquad [3.15]$$

where $D = \frac{|m' - I|}{\sigma'}$

Figure 3.2 - Loss function and ROV (NPV>0)



In this case, optimal decision is invest as *NPV* = *m'-I* > 0. Comparing the *NPV* with the value of this option, which is same as the cost of uncertainty, a similar criterion is reached:

Decision Criterion 2: (Case of NPV > 0)

ROV > |*NPV*| → Invest carefully

ROV < |*NPV*| → Invest

Rewriting this as:

Decision Criterion 2': (Case of NPV > 0)

$D < D^* \rightarrow$ Invest carefully

$D > D^* \rightarrow$ Invest

Defining the decision-making index $d \equiv (m'-I)/\sigma'$, which is given from eliminating the absolute value sign from *D*, these two decision criteria can be combined as:

Combined Decision Criterion

Invest	if	$D^* < d$
Invest carefully	if	$0 < d < D^*$
Wait and watch	if	$-D^* < d < 0$
Do not invest	if	$d < -D^*$
where $d = \frac{m'-I}{\sigma'}$,		D^* = 0.276

Consequently, only observing three parameters, m', σ' and *I* gives sufficient information to make more sophisticated decisions under uncertainty, expressing them in form of the new decision-making index, *d*.

Interpretation of *d* and *D**

How can *d* and *D** be interpreted? First, *d* can be seen as NPV divided by its standard deviation. In other words, *d* is the ratio of NPV to its uncertainty. Because *d* does not depend on the size of the project, it can be called "uncertainty-adjusted NPV" or "risk-normalized NPV". Several risky projects of which the sizes are different can easily compared. Next, when *d* = - *D**, option value to defer is equal to expected loss of NPV, namely, *D** is the break-even point of expectation of NPV and its option value, or the point where *ExNPV* = 0.

What is the probability that the payoff of this defer option is positive if *d* = - *D** at time 0? This can be calculated as follows,

$$P[V - I > 0] = P\left[\frac{V - I}{\sigma'} > 0\right]$$

$$= P\left[\frac{V - m' + m' - I}{\sigma'} > 0\right]$$

$$= P\left[\frac{V - m' + m' - I}{\sigma'} > 0\right]$$

$$= P\left[\frac{V - m'}{\sigma'} > -\frac{m' - I}{\sigma'}\right]$$

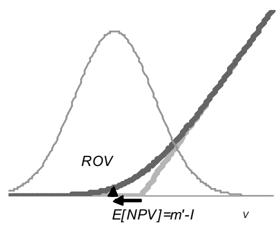
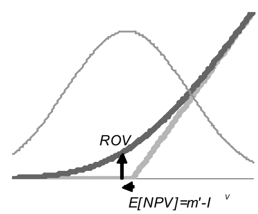
$$= P[N > -d]$$

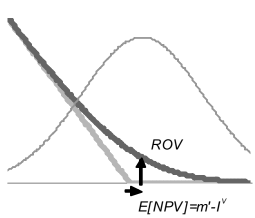
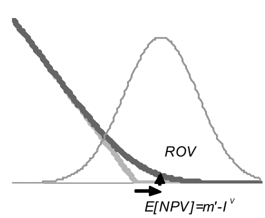
$$= P[N > D^*]$$

= .39

where *N* is standard normal distribution.

Figure 3.3 – Summary of the criterion

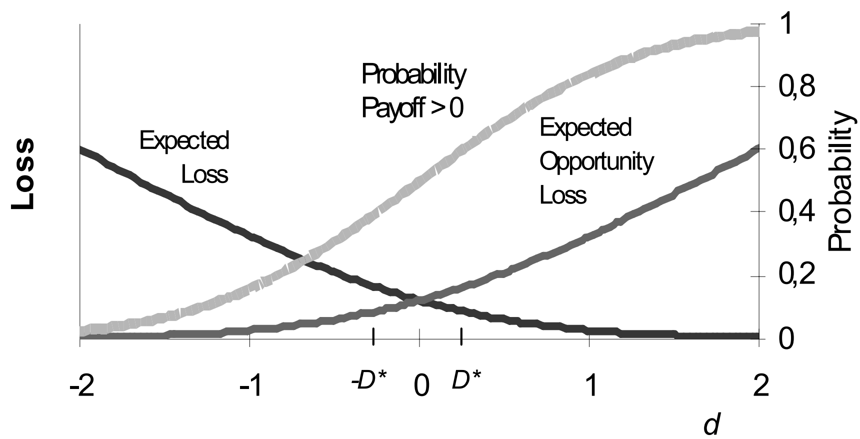
NPV	NPV < 0	
ROV	$\lvert NPV\rvert$ > ROV	$\lvert NPV\rvert$ < ROV
d	***d* < -D***	**-D* < *d* < 0**
Decision	***not Invest***	***wait and watch***
		

NPV	NPV > 0	
ROV	$\lvert NPV\rvert$ < ROV	$\lvert NPV\rvert$ > ROV
d	**0 < *d* < D***	**D* < *d***
Decision	***Invest carefully***	***Invest***
		

Because V is $N \sim (m', \sigma')$, $(V-m')/\sigma'$ is $N \sim (0, 1)$. Therefore, the probability that the payoff of the defer option is positive is 39 percent. Does it seem to be a high probability to abandon this option? Yes, it does! The criterion $d < -D^*$ means, "Do not invest now" but does not mean "abandon the defer option". The defer option itself has value though the expectation of NPV is deeply negative. If holding the option does not require any cost, it does not have to be thrown away! Just wait and watch what happens in the next period.

On the other hand, if $d = D^*$, the probability that the project will be out of the money is also 39 percent by symmetry. When the manager makes her decision to invest as $d = D^*$, there is still a probability of .39 of losing money. If the manager wanted a positive NPV with probability 90 percent, d should be higher than 1.28. It might be the case that the manager could set a higher d for the decision criterion if she would not care about opportunity losses. The tradeoffs between opportunity loss and cost of uncertainty are shown in Figure 3.4

Figure 3.4 – Tradeoffs of losses



The next section, shows an example of this criterion.

■ Example cases

Six independent projects

Consider the projects shown in Table 4.1. How can decisions be made using the new criterion? The assets that have current value S, time to expiration *T*, exercise price *K* at time *T*, volatility σ, and risk-free rate r_f. To calculate d, we have to solve *m'*, *I* and σ'. Assuming the value of S at time *T* is normally distributed with mean $S(1+ r_f)$T, *m'* = S because m' is expressed in present value. After setting $I = PV(K) = K/(1+ r_f)^T$, and, $\sigma' = S\sigma\sqrt{T}$, *d* = (*V*-*m'*)/ σ' can be calculated. Therefore, *d* can be calculated for each project and make decision to invest as shown in Table 4.1.

LUEHRMAN (1998a, 1998b) defined "option space" having two axes value-to-cost (*S*/*PV*(*K*)) and volatility ($\sigma\sqrt{T}$) and showed that decision criterion depends on the region in the option space. Using his example, similar results can be found with the new criterion. The new decision criterion is an integrated, simplified version of Luehrman's method.

Furthermore, project "E" in Table 4.1 is what was illustrated before in 2nd section. The new criterion gives the same decision, "wait and watch"!

Table 4.1 – Valuation for six independent projects

Variable	*A*	*B*	*C*	*D*	*E*	*F*
S	\$100.00	\$100.00	\$100.00	\$100.00	\$100.00	\$100.00
K_T	\$90.00	\$90.00	\$110.00	\$110.00	\$110.00	\$110.00
T	0.0	2.0	0.0	0.5	1.0	2.0
s	30%	30%	30%	20%	30%	40%
r_f	6%	6%	6%	6%	6%	6%
	\$0.00	\$42.43	\$0.00	\$14.14	\$30.00	\$56.57
S-PV(K)	\$10.00	\$19.90	-\$10.00	-\$6.84	-\$3.77	\$2.10
d	+infinite	0.469	-infinite	-0.484	-0.126	0.037
Exercice decision	Invest	Invest	Do not invest	Do not invest	Wait and watch	Invest carefully

S: current asset value
K_T: Exercise Price (at time = T)
T: Time to expiration (year)
s: Standard deviation of return (per year)
r_f: Risk-free rate of return (% per year)

Conclusion

Applying real option valuation methodology shows that the new decision index d – the uncertainty adjusted NPV – and D^* = 0.276 – the break-even point of NPV and ROV (real option value) – gives a clear solution to make a decision under uncertainty. When making decision, managers have to observe only three parameters: expectation of future cash flow, its uncertainty, and the amount of investment to acquire the project. The examples using the new criterion show its usefulness.

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